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NON-LINEAR WAVES AND QUASI-ORDERED STRUCTURES IN TURBULENCE.(U)

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The research seeks a deterministic analytical model, derived from first principles without empiricism, for the development and evolution of quasi-ordered, large scale, events/structures in turbulent flows. Upon a careful analysis of recent experiments with homogeneous incompressible transitional and turbulent mixing layers, the events/structures are associated with non-linear instability waves whose behavior is dominated by seld-induced effects rather than by interactions among instabilities which are resonant according to the linear dispersion relation. The non-linear transport of vorticity in the neighborhood of

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the critical layer is identified as the flow process responsible for the self-induced effects beginning at modest wave amplitudes A $\gtrsim 10^{-2}$. A mathematical model of the attendant generation of bound-wave harmonics, subharmonics and sideband frequencies in strictly two-dimensional incompressible flow subject to monochromatic disturbance input is developed by joint use of matched asymptotic expansion and stretched coordinates techniques. The predictions of the model are compared favorably with relevant experimental data for mixing layers. Model extensions required for the analysis of modulated three-dimensional non-linear disturbances in non-homogeneous compressible flows are outlined, and the role of those results in describing the evolution of quasi-ordered events/structures in practical turbulent flows are discussed.

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INTERIM REPORT
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NON-LINEAR WAVES AND QUASI-ORDERED STRUCTURES IN TURBULENCE

NEW YORK UNIVERSITY
DEPARTMENT OF APPLIED SCIENCE
NEW YORK, NEW YORK 10003

ROBERTO VAGLIO-LAURIN

Report Date - October 1978

I. RESEARCH OBJECTIVES

The premise of the theoretical research effort under the title contract resides in the recent experimental evidence about the presence in all turbulent flows of quasi-ordered, large scale events/structures, which occur randomly, but with statistically definable mean periods (Ref. 1). The nature, growth, saturation and regeneration of the structures vary from family to family of flows, i.e. according to whether a two-dimensional mixing layer, or a two-dimensional jet or a two-dimensional boundary layer are observed. However, for a given family of flows, the features and history of the structures show little dependance on Reynolds number throughout the transitional and the fully developed turbulent regimes. In both regimes the evolving structures appear to dominate the macroscopic aspects of the flow, including its rate of growth/entrainment and the associated Reynolds stresses; in addition, the process of intermittent turbulence production by three-dimensional energy cascade appears to be connected with high wave-number instabilities evolved within, and amplified by, the structures themselves.

These findings suggest a unified view of the turbulent process, namely: in any specific family of flows, the dynamics of turbulence is paced by the inviscid interaction between the mean flow and a characteristic space-dependent non-linear fundamental large scale mode, which develops in deterministic fashion and, in the process, intrinsically evolves deterministic conditions for its own periodic extinction and regeneration. The research under the title contract aims to concretely validate that view through the four-fold objective of 1) identifying the basic flow processes which underlie the development of the non-linear large scale mode characteristic of each family of flows, 2) constructing from first principles an attendant theoretical model descriptive of the mode's cyclic history and interactions with the mean flow, 3) determining whether this history can be altered by perturbations imposed either on the flow or on its boundary conditions and, consequently 4) exploring possibilities for external control of transition and turbulent flow development which may lead, for example, to reduced skin friction drag of flight vehicles, optimized fuel/air mixing in engine burners under a variety of operating conditions, improved optical quality of the gas flow through chemical laser cavities, etc.

Underlying the research objectives 1 and 2 is the conjecture that a single flow process may be responsible for the non-linear cyclic development of the large scale structures in all families of flows and, thus, a single mathematical model, derived from first principles without empiricism, may be adequate for their description. The effort carried out during the first year of research, covered by the present report, has succeded in 1) identifying the basic flow process as the non-linear transport of vorticity in the neighborhood of the critical layer for periodic flow perturbations (waves) possessing small but finite amplitude, and 2) developing the attendent mathematical model for the case of strictly two-dimensional waves. The data analyses and model formulation which support and embody this statement of progress are described in the following section of the report.

II. STATUS OF THE RESEARCH EFFORT

The effort under Contract F49620-77-C-0-119 was initiated with a careful review and study of recent experimental data for turbulent incompressible homogeneous mixing layers. The salient features of these data include:

a) a fundamental periodicity of the velocity fluctuations consistent with a similarity scaling throughout the regions of non-linear transitional and fully turbulent flow regimes (Refs. 2, 3);

b) a remarkable agreement between the fundamental periods corresponding to measurements at the low-and high-speed edges of the mixing layer (Ref. 2);

c) a significant Reynolds stress production associated with the pairing interactions in which two large scale structures combine to form a single, larger one (Ref. 4);

d) two distinct and separate stages of external fluid entrainment and fine scale mixing, the former being associated with the irrotational large scale motion of the large structures during their individual life-time, and the latter being correlated with the disappearance of any particular large structure upon pairing interaction (Refs. 2, 3);

e) surprisingly long correlation times associated with the large-structure dynamics.

The features a and b strongly suggest that wave-like, large scale structures are generated repetitively with self preserving characteristics, except for the length scale, which increases downstream like the local shear layer width. From a wave mechanics viewpoint this implies a repetitive process whereby a wave of frequency β generates a subharmonic of frequency $\beta/2$, which then grows to become dominant and to generate its own subharmonic, at an essentially deterministic self-preserving rate. In addition to being consistent with the visually observed pairing interaction, the process of subharmonic generation at large scale appears to possess a surprisingly large influence upon the stress in, and the growth of, the mixing layer according to the features noted at c and d above. The generation of smaller scale turbulence appears to be intrinsic to, and responsible for, the extinction of individual large structures upon their pairing (see d above). Although fine scale mixing results from this process, little influence on overall flow development is indicated. Thus, a model descriptive of the large structure dynamics should yield, as a by product, the main features of turbulent flow development.

The nature and dynamics of large structures outlined above, as well as the visual observations of Refs. 2 through 6, may be rationalized from a dual viewpoint, viz either in terms of Stuart vortices which agglomerate (e.g. Ref. 4), or in terms of non-linear waves. Whereas the vortex approach has been found to be only qualitatively successful (Ref. 4) and incapable of reproducing the observed long correlation times (feature e), the wave-mechanical viewpoint was adopted for the investigations described here.

A wave-mechanical viewpoint immediately suggests a model formulation within the framework of classical linear and weakly non-linear stability theories. However, a comparative examination of available theoretical and experimental results (e.g. Refs. 7, 8) readily reveals the inadequacies of the classical stability theories in accounting even for the early stages of transition in the flow. On the theoretical side, it is found that predictions for linear spatially growing waves (which initiate the transition process in a shear layer) do not yield conservation of vorticity and, therefore, violate the very equation whose eigen-solutions have presumably been determined. In the example of Fig. 1, vorticity peaks of magnitude -0.8 are calculated within two wave-lengths of the origin for a hyperbolic tangent mean velocity profile having maximum vorticity -0.5 (Ref. 7). On the experimental side, harmonically resolved measurements of r.m.s. velocities (Fig. 2, Ref. 8) indicate the presence of harmonics and subharmonics which grow at rates quite different from those predicted either by linear theory (Fig. 3, Ref. 7) or by weakly non-linear theory*. In fact, neither theory is capable of justifying either the presence of the subharmonics or their initial equilibration simultaneous with that of the fundamental (Fig. 2), even if appeal is made to hypothetical resonances, which are strictly possible only in temporally (as opposed to spatially) growing instabilities. Thus, theory and experiments combine to suggest that: a) the linear stability results are not uniformly valid; b) recognition of some localized effect is required to restore uniform validity to the theory; c) the predicated effect must presumably be non-linear if it is also to account for the generation of harmonics and subharmonics

* The initial growth rates of the harmonics and the subharmonic in Fig. 2 are, respectively, 1.5 times and 1.25 times that of the fundamental.

having the behavior observed in the experiments (e.g., Fig. 2).

The source of the nonuniform validity in the linear stability solutions becomes readily apparent upon an examination of the vorticity conservation equation. For the strictly two-dimensional problem considered here, i.e. a basic parallel flow with velocity ($\bar{U}$, 0, 0), vorticity (0, 0, Ω) and Reynolds number Re (based on average mean flow velocity $\bar{U}$ and flow width L), plus a superposed perturbation of small amplitude A involving velocities (u, v, 0) and vorticity (0, 0, ω), the equation is

$$\frac{\partial \omega}{\partial t} + (\bar{U} - c) \frac{\partial \omega}{\partial x} + v \frac{\partial \Omega}{\partial y} = - A \left(u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y}\right) + \frac{1}{A\,Re} \nabla (\Omega + A\omega) \qquad (1)$$

in a frame of coordinates moving with the phase velocity c of the disturbance. Whereas the solutions of classical linear inviscid stability theory are obtained by setting to zero the right hand side of equation (1), the approximation must fail whenever one of the terms retained at the left hand side goes to zero, viz. for either $(U - c) \rightarrow 0$ near the critical point ($y = y_c$), or for $(\partial\Omega/\partial y) \rightarrow 0$ near the inflection point. In those neighborhoods the correct vorticity balance must take into account either one of the neglected terms on the right hand side of equation (1). A uniformly valid solution to equation (1) must accordingly be evolved in the context of classical matched asymptotic expansion techniques, with the expansion parameter selected in accordance with the predominant term retained on the right hand side of the equation.

The classical approach to linear and weakly non-linear stability theories is predicated on the assumption that, for high Reynolds number flows such as those of interest here, the singular nature of the linearized inviscid perturbation equation (Rayleigh equation) in the neighborhood of the critical layer is removed at all wave amplitudes by the sole effect of vorticity diffusion as embodied in the linearized viscous perturbation equation (Orr-Sommerfeld equation). This predominant effect of viscosity independent of wave amplitude is challenged by the substantive experience accumulated with singular perturbation problems. On that basis it may be argued that, depending upon wave amplitude, the singularity of the linearized inviscid perturbation equation is removed by either the effect of the small linear higher order viscous operator or the effect of the small non-linear operator descriptive of non-linear vorticity convection in two-dimensional wave motions and of vorticity convection as well as stretching in three-dimensional wave motions. Whereas the viscous effect is confined to a (boundary) region about the critical layer having thickness independent of wave amplitude and proportional to $(\alpha R)^{-1/3}$ (where R denotes the Reynolds number of the mean flow and α the dimensionless wave number of the disturbance), while the non-linear effects influence the flow in a region about the critical layer having thickness proportional to $(A/u_c')^{\frac{1}{2}}$ (where A denotes the disturbance amplitude and u_c' the dimensionless mean velocity gradient at the critical layer), there is always a wave amplitude A_c wherefor the boundary layer scale associated with the non-linear effect becomes larger than the scale associated with the viscous effect. For $A \gtrsim A_c$ the Orr-Sommerfeld description of the wave motion near the critical layer ceases to be valid, and a new class of asymptotic solutions to the Navier Stokes equations, recognizing the non-linear vorticity convection and stretching, are required to describe the subsequent growth and/or decay of the wave.

Standard considerations of boundary layer scale readily show that the term $O\,(Re^{-1})$ at the right hand side of (1) predominates whenever $A \lesssim 10^{-2}$, while the term $O(A)$ takes over for $A \gtrsim 10^{-2}$. Our interest being focussed on the non-linear transitional and fully turbulent flow regimes (where typically $A \simeq 10^{-1}$), detailed model development must be carried out by retaining the lateral perturbation vorticity transport, $O(A)$, in equation (1). The matched asymptotic expansion technique then shows that, for a spatially growing wave of amplitude A, wave number α_r, growth rate $(-\alpha_i)$ and frequency β, the noted non-linear term contributes measurably to the vorticity balance within an (inner) region of lateral extent $(\delta/L) = O[A_o^{\frac{1}{2}} \exp(-\frac{\alpha_i}{2} x)] = O\,[A^{\frac{1}{2}}]$ about the critical point, provided the wave amplitude is large enough to render the parameter $\delta = |\beta\alpha_i\, \alpha_r^{-2} (u_c' A)^{-\frac{1}{2}}| < 1$. Under those conditions the flow obtained by the superposition of a small, but finite, amplitude two-dimensional wave upon a basic parallel flow, is characterized by a stream function ψ in wave fixed coordinates having matched inner and outer expansions, respectively, of the form

$$\psi^{(i)}\,(x,y,t) = \sum_{m=0,1,..} A^{1+(m/2)}\, \Phi_m\left[\xi,\, \psi \exp\left(-\frac{\alpha_i}{\alpha_r}\,\xi\right),\, \delta\right] \tag{2a}$$

$$\psi^{(o)}\,(x,y,t) = \int_{y_c}^{y} (\bar{U}-c)\, dy + \sum_{n=0,1..} A^{1+(n/2)}\, \omega_n\left[y,\, \xi,\, \delta\right] \tag{2b}$$

where

$$\xi = (\alpha_r\, x - \beta t) \tag{3a}$$

$$\psi = \left[\frac{u'_c}{A} \frac{(y - y_c)^2}{2} + \cos \xi \right] \tag{3b}$$

The dependence of the functions Φ_m and φ_m upon the parameter $\delta < 1$ in equations (2a,b) does not provide a uniformly valid description of the flow and, thus, must be removed. This is done by applying Lighthill's method of stretched coordinates to the inner expansion to obtain

$$\zeta + \sum_{p=1,2,..} \delta^p \zeta_p (\xi,\zeta) = \left[\frac{u'_c}{A} \frac{(y-y_c)^2}{2} + \cos \xi\right] \tag{4a}$$

$$\Phi_m \left[\xi, \psi \exp\left(-\frac{\alpha_i}{\alpha_r} \xi\right), \delta\right] = \sum_{p=0,1,..} \delta^p \Phi_{mp} \left[\xi, \zeta \exp\left(-\frac{\alpha_i}{\alpha_r} \xi\right)\right] \tag{4b}$$

the functions Φ_{mp} being required to be periodic in ξ. Detailed matching in the overlap domain $(y-y_c) \to 0$, $\zeta \to \infty$ then yields

$$\varphi_n \left[y, \xi, \delta\right] = \sum_{p=0,1,..} \delta^{p/2} \varphi_{np} \left[y,\xi\right]$$

$$= \sum_{p=0,1,..} \sum_{q=o,1,..} \delta^{p/2} \varphi_{npq} \left[y\right] \cos \left(q \frac{\xi}{2}\right) \tag{4c}$$

By this process the effect of non-linearity (viz, transversal transport of perturbation vorticity near the critical layer) in forcing mean flow perturbations, harmonics and subharmonics in the outer region becomes manifest.

Specifically, one finds that

TABLE I

INNER SOLUTIONS		FORCE	OUTER SOLUTIONS	
m	p		n	q
0	0		0	2
1	0		0	2
2	0		{ 0	0
			1	0,2,4,...
2	1		1	1
2	2		1	3

i.e. non-linearity results in harmonics, subharmonic and 3/2- harmonic having growth rates in agreement with the experimental observations of Fig. 2. In addition there is a spatially amplifying mean flow perturbation ($n = 0$, $q = 0$), which, together with the subharmonic, appears to provide the key to the subsequent flow development as outlined in the following paragraphs.

According to Fig. 2 the fundamental, harmonics and subharmonics grow at their respective rates until they all equilibrate simultaneously at $x \approx 7$cm. This behavior is consistent with the predictions of the model described above; clearly, the fundamental provides the driving mechanism throughout the considered interval for simultaneous equilibration of the harmonics and subharmonic to be achieved. Whereas the experimental data suggest that the equilibration is well described by a Landau type equation, one may surmise that the attendant mathematical description may be extracted from the present, matched asymptotic expansion, non-linear wave model by allowing the attendant component solutions to depend upon a slow varying spatial variable, viz. $X = x\delta^{-1}$. Analogy with the results of weakly non-linear theory (Ref. 9) then suggests that the condition of integrability for the n = 2 outer solution should yield the desired equation.

Subsequent to the equilibration of the fundamental two significant effects arise according to the data of Fig. 2, namely: 1) the subharmonic, of dimensionless frequency ($\beta/2$) in the flow of width L, first resumes growth at the rate appropriate to a linear wave of dimensionless frequency β in a mean flow of width 2L, and then equilibrates at essentially the same amplitude as did the original fundamental; 2) the original fundamental decays in the presence of three-dimensional distortions and formation of secondary streamwise vortex structures. Again the model of equations (4-a,b,c) appears to provide the framework for rational interpretation and deterministic analysis of these events. Specifically, one may surmise that the first effect, which esentially describes the repetitive agglomeration of large scale structure typically observed in turbulent mixing layers, develops at the station where the lateral scale of the mean flow [which according to Table I is forced to grow at a rate $(-\alpha_i)$] matches the frequency/wave number of the subharmonic, so that the latter becomes an eigensolution of the problem. The second effect may be interpreted either as the onset of Taylor instabilities within certain portions of the finite amplitude large-scale structure associated with the initially dominant two-dimensional instability, or as the effectively equivalent development of non-linear three-dimensional motions of the type observed in boundary layer transition experiments (Ref. 10), triggered by a spanwise modulation in the amplitude of the dominant two-dimensional instability.* In either case streamwise vortices are generated which, in turn, must stretch and precess around one another, eventually breaking down into smaller scale structures. Thus, quasi-deterministic processes of intermittent energy cascade to high wave numbers and attendant turbulence generation can be envisaged.

The key points in this deterministic view of the cyclic history of two-dimensional large scale structures are: a) unstable modes exhibit non-linear behavior beginning at modest amplitudes $A \gtrsim 10^{-2}$; b) the non-linear behavior at $A \gtrsim 10^{-2}$ forces simultaneously a larger scale subharmonic mode (the seed of the next structure) and a mean flow perturbation growing at distinct rates; c) as the initially dominant mode equilibrates the perturbed mean flow and the forced subharmonic become geared to "resonate" and, thus, to provide growth of the next structure; d) while the new structure grows, the initial one decays by cascading energy to three-dimensional modes which represent instabilities intrinsic to the non-linear vorticity distribution within the initial structure itself; e) only the model of equations (4-a,b,c) appears capable of describing the non-linear effects noted at b), clearly prerequisite to the statistically

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* An analysis of the data of Ref. 10, carried out under a parallel OSR-sponsored effort, suggests that non-linear transport and stretching of vorticity near the critical layer (i.e. the very flow processes addressed by the present model) govern the non-linear phase of boundary layer transition.

periodic history of events indicated at c) and d) and observed in experiments.

The fact that the model of equations (4-a,b,c) reproduces so many features of the non-linear phases of mixing layer transition, as observed in the specific experiment of Ref. 8 (Fig. 2), is quite gratifying. However, since the model as well as the experiment are concerned with situations wherein the spectrum of input disturbances is sharply peaked about a single frequency, one may wonder about the applicability/extension of the results to actual transitional and turbulent flows, which are characterized by relatively broad spectra of input disturbances. The considered effects and the model are non-linear; thus, the principle of superposition may not be applied to the study of distinct, dispersive, finite amplitude waves having either coincident or overlapping critical layer regions. By contrast, superposition may be invoked if the considered waves are bound (i.e. the wave train possesses non-dispersive characteristics), for then the flow processes within the critical layer region retain coherence throughout the non-linear development of the wave/structure. Given an input spectrum of disturbances which are dispersive in the linear limit, the first situation can be expected to prevail if the effects of randomness dominate over the effects of non-linearity, while the second situation may be anticipated in cases where the non-linearity becomes controlling. Given the ample experimental evidence about quasi-ordered structures with statistically definable characteristics, we submit that turbulence dynamics is controlled by the effects of non-linearity, with the effects of randomness entering only at the next order of approximation. Accordingly, the model of equations

(4-a,b,c) embodies the basic features of the attendant processes. This view, which represents the second and perhaps predominant departure of the present research effort from the classical trends of classical weakly non-linear stability theory,* is justified in preliminary fashion by the arguments of the following paragraphs.

The development of a coherent structure in a turbulent flow is conceptually equivalent to the development of the predominant non-linear wind-driven wave on the free surface of a body of water. In both cases linear wave trains (dispersive in the linear limit) evolve, through non-linear processes, into non-linear wave trains possessing broad power-spectra characterized by a predominant frequency. A comparative study of related experiments can then be instructive. The results of Reference 13 prove quite revealing as they show that, under conditions of fixed fetch and steady wind blowing in one direction: 1) essentially all the energy in the resulting non-linear wind-wave system is contained in the bound wave components of a single dominant wave of frequency ω_o, wave number k_o; 2) the individual spectral components do not propagate as free waves and do not obey the usual dispersion relation; 3) energy is propagated at a single group velocity corresponding to the dominant frequency; 4) the evolution

* Weakly non-linear stability theory is predicated on the assumptions that: 1) to a first approximation disturbance spectra are made up of many linear, random, free wave components with Gaussian or near-Gaussian statistics; 2) non-linear interactions are effective only among wave components which are resonant based on the linear dispersion relation for free waves.

of the wave train is well described by the non-linear Schrödinger equation

$$i\left(\frac{\partial A}{\partial t} + \frac{\omega_o}{2k_o}\frac{\partial A}{\partial x}\right) - \frac{\omega_o}{8k_o^2}\frac{\partial^2 A}{\partial x^2} - \frac{\omega_o}{2}k_o^2 |A|^2 A = 0 \qquad (5)$$

where A denotes the complex envelope of the wave train related to the free surface displacement $\eta(x,t)$ through

$$A(x,t) = \alpha(x,t) \exp\left[i\, \theta\, (x,t)\right] \qquad (6a)$$

$$\eta(x,t) = \alpha(x,t) \cos\left[(k_o x - \omega_o t) + \theta(x,t)\right] \qquad (6b)$$

with $\alpha(x,t)$ and $\theta(x,t)$ slowly varying functions of x and t.

It is quite interesting that equation (5) [or close equivalents thereof] has been obtained in analytical studies of the long time evolution of Stokes wave trains subject to modulational perturbations (Refs. 14, 15, 16)* as well as a generalization of Landau's equation for near neutral non-linear wave systems in parallel shear flows (Refs. 17, 18). Thus, experiments for wind waves and theory for those waves as well as for shear flow instabilities are congruent provided the flow mechanism is credited with a narrow band-pass capability which selects a specific dominant frequency at the considered fetch.

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* A Stokes wave train is characterized by a single fundamental frequency, a finite but uniform amplitude, and a power spectrum with a dominant component at the carrier frequency plus a series of less energetic components at the frequencies of the bound-wave harmonics of the carrier. Such a wave train is unstable to modulational perturbations.

As is well known, the dominant frequency of a wind wave system changes (diminishes) with fetch, so one must ask how energy is shifted from one carrier frequency to the next lower one and, thus, how the narrow band-pass characteristic is achieved. Again the experiments of Ref. 13 prove quite illuminating by showing that, even in the absence of wind, a modulated non-linear wave train can undergo a self-induced shift to a new lower carrier frequency whenever further growth of the modulation would require some wave to exceed a maximum realizable steepness. If the modulation frequency $\Delta\omega_o$ is prescribed, the frequency shift is from ω_o to $(\omega_o - \Delta\omega_o)$ i.e. from the dominant frequency of the initial wave-train to the lower of the pair of sideband frequency components representing the amplitude modulation. In wind-driven waves $\Delta = ak_o$, where a denotes the wave amplitude and k_o wavenumber, in accord with the theory of Refs. 14 and 15; however, no theory exists for predicting the conditions of amplitude and modulation under which the frequency shift occurs. The development of such a theory would allow the rate of occurrence of frequency shift to become predictable and, accordingly, the evolution of wave spectra to become amenable to deterministic description.

The direct analogy between the findings of Ref. 13 and the experimental evidence for turbulent shear flows resides in the observation that: 1) at a given distance (fetch) from the origin of the flow, equivalent roles are played by the dominant wave and by the quasi-ordered structure; 2) statistically unique frequency/scale of the dominant structure/wave at that fetch are determined by the upstream sequence of selective frequency shifts allowed by the self-induced non-linear behavior of the locally dominant wave. Whereas the model of equations (4a,b,c) describes the non-linear wave behavior in strictly two-dimensional cases, identifies the generation of attendant sideband frequency (subharmonic) as well as mean flow perturbations, and finally, upon non-linear equilibration of the carrier, allows for the growth of the sideband frequency mode consistent with the observations of Refs. 2 through 6 and 8, the advocated analogy between wind-driven waves and shear flow instabilities appears to encompass all aspects of the dominant instability evolution in the two problems. In this context the model of equations (4a,b,c) provides for two-dimensional shear flow instabilities the closing theoretical link, which is still missing in the wind-wave problem. Its significance vis-a-vis the classical viewpoint of weakly non-linear stability theory mainly resides in its ability to characterize the salient features of experimental observations in transitional and turbulent flows, viz. the generation/growth of sideband frequency modes (subharmonics in the case of two-dimensional mixing layers) and the selection of a fetch dependent frequency for the dominant fluctuation mode, as inherent, deterministic, self-induced effects of non-linear wave behavior.

To be sure the description of flow more complicated than the two-dimensional incompressible mixing layer requires that the model of equations (4a,b,c) be extended to describe the non-linear behavior of 1) two-dimensional waves possessing periodic spanwise amplitude modulation in incompressible flows and 2) oblique as well as two-dimensional waves with and without spanwise periodic amplitude modulation in compressible flows. A comparison between the self-induced non-linear effects determined by these analyses and the experimental evidence about quasi-ordered events should then reveal the dominant mechanism for transition and turbulence sustenance in each family of flows. According to the discussions above this line of inquiry is the most promising one; thus, the continuation of the research effort is to be focused on it.

III. PROFESSIONAL PERSONNEL ASSOCIATED WITH THE RESEARCH EFFORT

In addition to the principal investigator Dr. Roberto Vaglio-Laurin, Professor of Applied Science, the following graduate students/research assistants have participated in the research under Contract F49620-77-C-0-119 during the year 1 July 1977 through June 30, 1978: C.L. Chou, J.P. Clesca, C.T. Hsieh, T.K. Sengupta, Y. Wey.

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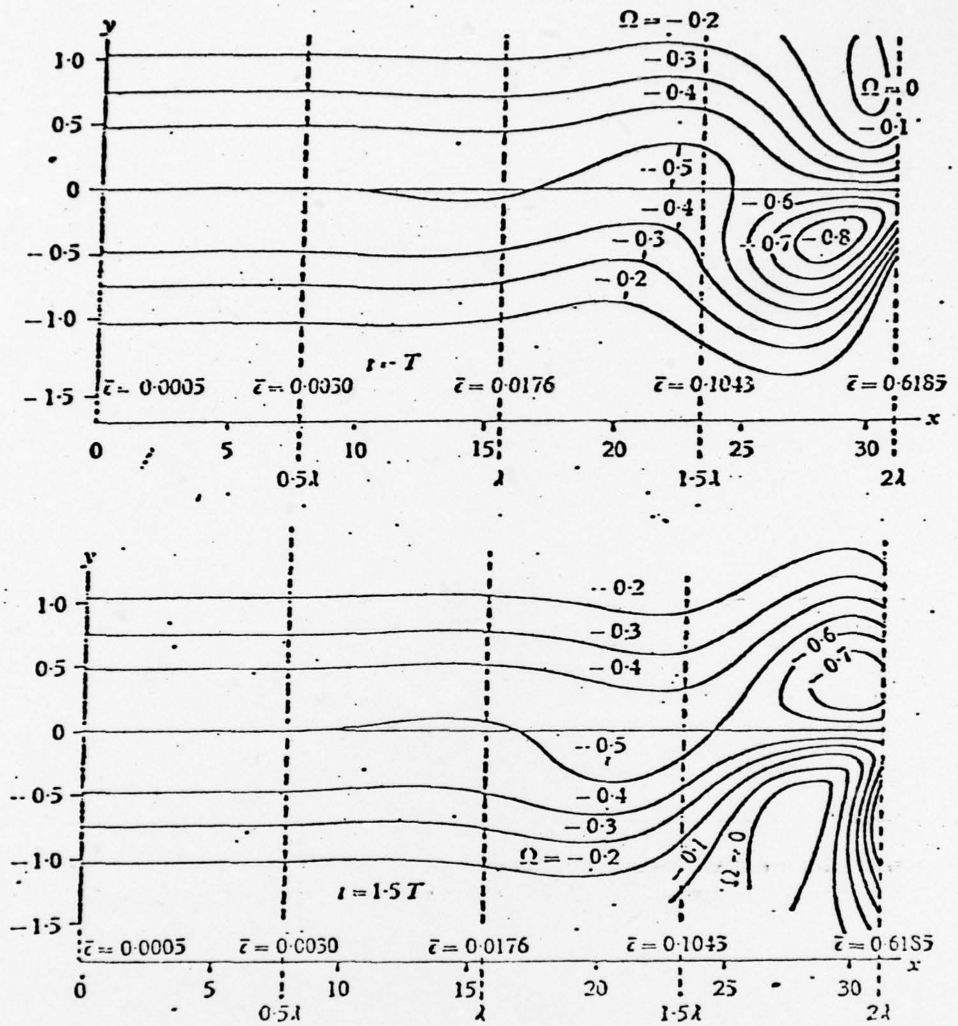

Fig. 1. Spatially growing disturbances in a free shear layer. Lines of constant vorticity predicted by linear stability theory. After Michalke (Ref. 7).

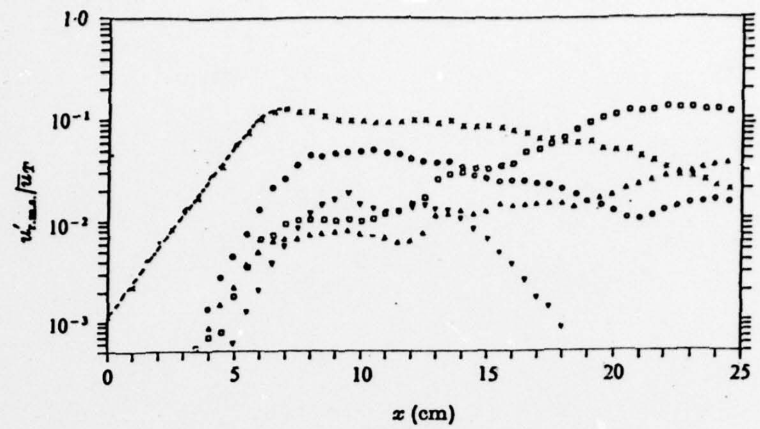

Fig. 2. Observed spatial growth of harmonically resolved maximum rms. velocity in the non-linear stages of free shear layer transition. x, fundamental frequency β, o, 2β, ∇, 3β, $\square$, $(\beta/2)$; Δ, $(2\beta/2)$. After Miksad (Ref. 8).

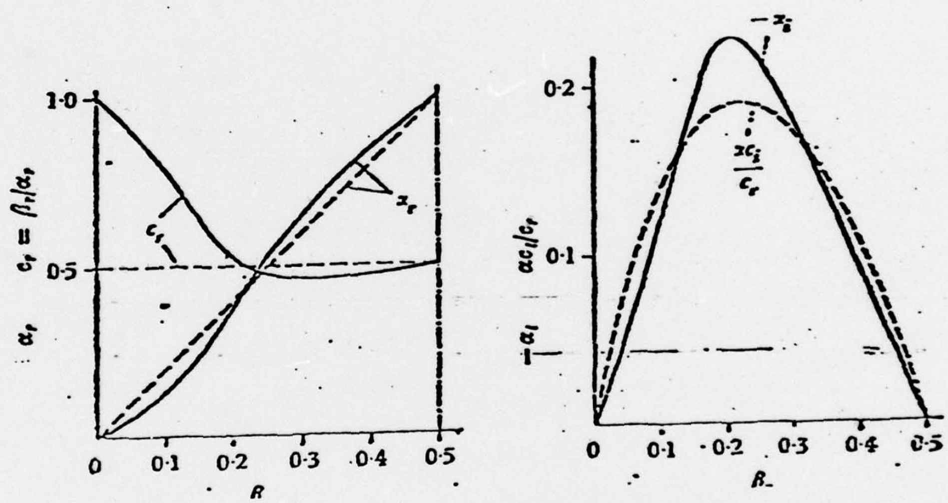

Fig. 3. Spatially (-) and temporally (---) growing disturbances in a free shear layer. Predictions of linear stability theory for wave number α_r, phase velocity c_r, and spatial amplification rate α_i as a function of frequency β_r. After Michalke (Ref. 7).